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ON UNIVEXITY-TYPE NONLINEAR PROGRAMMING PROBLEMS IN COMPLEX SPACES

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Abstract :

In this paper , we will study a new class of nonlinear programming called SFJ-univex introduced in [13] in complex spaces , combining the concepts of SFJ-invex programming and univex functions in complex spaces. Optimality and duality results for several mathematical programs are obtained under the above –mentioned assumption .

1.Introduction :

Consider the following nonlinear programming problem :

$$(p) \quad \min \operatorname{Re} f[z, \bar{z}, w, \bar{w}]$$



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such that $\operatorname{Re} g[z, \bar{z}, w, \bar{w}] \leq 0$, $[z, \bar{z}, w, \bar{w}] \in C^n \times C^n \times C^m \times C^m$

where $f: C^n \times C^n \times C^m \times C^m \rightarrow C$, $g: C^n \times C^n \times C^m \times C^m \rightarrow C$.

Several classes of functions have been defined for the purpose of weakening the limitations of convexity in the mathematical programming problem (P).

Xu[16] proposed the new class of nonlinear programming, called SFJ-invex programming, which lies between invex programming and type I programming [3,6,7].

Bector et al. [2] introduced the concept of univexity as a generalization of convexity. Recently, Rueda et al. [10] introduced a new class of functions, combining the concepts of type I and univex functions and obtained optimality and duality results for several mathematical programs. Then further S.K.Mishra and N.G.Rueda [13] introduced and discussed SFJ-univex programming problems. This paper can be viewed as an extension of [13] in complex spaces.

Again after enhancement of these areas after introduction in complex spaces these areas become wider than a lot of consequences of papers have been published yet, some of them are [1],[4],[5],[8],[11],[14],[15].

2. Preliminaries:



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In this paper , we introduced the SFJ-univex functions in complex spaces and proposed the following .

To compare vectors along the lines of Mangasarian , we will distinguish between \leq and \leq or \geq and \geq , specifically ,in complex space . Let For C^n denote an n-dimensional complex spaces .For $z \in C^n$, let the real vectors $\text{Re}(z)$ and $\text{Im}(z)$ denote the real and imaginary parts , respectively , and let $\bar{z} = \text{Re}(z) - i\text{Im}(z)$ be the conjugate of z . Given a matrix $A = [a_{ij}] \in C^{m \times n}$, where $C^{m \times n}$ is the collection of $m \times n$ complex matrices , let $\bar{A} = [\bar{a}_{ij}] \in C^{m \times n}$ denote its conjugate matrix , let $A^H = [\bar{a}_{ij}]$ denote its conjugate transpose . The inner product of $x, y \in C^n$ is $(x, y) = y^H x$.Let R_+ denote the half line $[0, \infty[$.

$z \in C^n$, $v \in C^n$, $\text{Re}(z) \leq \text{Re}(v) \Leftrightarrow \text{Re}(z_i) \leq \text{Re}(v_i)$, for all $i = 1, 2, \dots, n$, $\text{Re}(z) \neq \text{Re}(v)$.

Similar notations are applied to distinguish between \geq and \geq .

For a complex function $f : C^n \times C^n \times C^m \times C^m \rightarrow C$ analytic with respect to $\zeta = (w^1, w^2)$,

$z \in C^n$, define gradients by



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$$\nabla_z f_i(v, \bar{v}, \zeta) = \left[\frac{\partial f}{\partial w_i^1}(v, \bar{v}, \zeta) \right] \quad i=1,2,\dots,n.$$

$$\nabla_{\bar{z}} f_i(v, \bar{v}, \zeta) = \left[\frac{\partial f}{\partial w_i^2}(v, \bar{v}, \zeta) \right] \quad i=1,2,\dots,n.$$

Definition 2.1:

The Problem (P) is said to be SFJ-univex if there exist $\eta : C^n \times C^n \times C^m \times C^m \rightarrow C$,

$$\phi_0 : C \rightarrow C, \phi_i : C \rightarrow C, i=1,2,\dots,m, b_0 : C^n \times C^n \times C^m \times C^m \rightarrow C_+,$$

$$b_i : C^n \times C^n \times C^m \times C^m \rightarrow C_+, i=1,2,\dots,k.$$

Such that

$$\left. \begin{array}{l} (z_0, \bar{z}_0) \in C^n \times C^n \\ \\ (z, \bar{z}) \in C^n \times C^n \end{array} \right\} \left\{ \begin{array}{l} \text{Re}[b_0(z, \bar{z}, z_0, \bar{z}_0) \{ \phi_0 \{ f(z, z, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w}) \} \}] \\ \geq \text{Re}[\nabla_z f(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0) \\ + \nabla_{\bar{z}} f(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)] \\ \\ \text{If } \text{Re}[g_i(z, z, w, \bar{w})] = 0, i=1,2,\dots,k. \text{ then} \end{array} \right. \quad (1)$$



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$$\begin{aligned}
 & \text{Re}[b_i(z, \bar{z}, z_0, \bar{z}_0)\{\varphi_i\{g_i(z, z, w, \bar{w})\}\}] \\
 & \geq \text{Re}[\overline{\nabla_z g(z_0, \bar{z}_0, w, \bar{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)} \\
 & + \nabla_{\bar{z}} g(z_0, \bar{z}_0, w, \bar{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)}] \tag{2}
 \end{aligned}$$

Definition 2.2:

The Problem (P) is said to be SFJ-invex if there exist $\eta : C^n \times C^n \times C^m \times C^m \rightarrow C$, Such that

$$\left. \begin{array}{l} (z_0, \bar{z}_0) \in C^n \times C^n \\ \\ (z, \bar{z}) \in C^n \times C^n \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \text{Re}[f(z, z, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})] \\ \geq \text{Re}[\overline{\nabla_z f(z_0, \bar{z}_0, w, \bar{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)} \\ + \nabla_{\bar{z}} f(z_0, \bar{z}_0, w, \bar{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)}] \tag{3} \\ \\ \text{If } \text{Re}[g_i(z, z, w, \bar{w})] = 0, i = 1, 2, \dots, k. \text{ then} \\ \\ \text{Re}[g_i(z, z, w, \bar{w})] \\ \geq \text{Re}[\overline{\nabla_z g_i(z_0, \bar{z}_0, w, \bar{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)} \\ + \nabla_{\bar{z}} g_i(z_0, \bar{z}_0, w, \bar{w})\eta^T(z, \bar{z}, z_0, \bar{z}_0)}] \tag{4} \end{array} \right.$$



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Theorem 2.1:

Every SFJ-invex program is an SFJ-univex program, but the converse may not be true.

Proof:

Assume that a given program (P) is SFJ-invex, i.e., there exists a function

$\eta : C^n \times C^n \times C^m \times C^m \rightarrow C$, Such that

$$\left. \begin{array}{l} (z_0, \bar{z}_0) \in C^n \times C^n \\ (z, \bar{z}) \in C^n \times C^n \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \text{Re}[f(z, z, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})] \\ \geq \text{Re}[\overline{\nabla_z f(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)} \\ + \nabla_{\bar{z}} f(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)] \\ \\ \text{If } \text{Re}[g_i(z, z, w, \bar{w})] = 0, i = 1, 2, \dots, k. \text{ then} \\ \\ \text{Re}[g_i(z, z, w, \bar{w})] \\ \geq \text{Re}[\overline{\nabla_z g_i(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)} \\ + \nabla_{\bar{z}} g_i(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)] \end{array} \right.$$

Hence, problem (P) is SFJ-univex with respect to $b_0 = b_i = 1$,

$\text{Re}[\phi_0(f)] = \text{Re}[f]$, $\text{Re}[\phi_i(g)] = \text{Re}[g_i]$, for $i=1, 2, \dots, k$ and the same η .

For the converse part see the following example :

Example 2.1:



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Consider the following problem :

$$\min f[z, \bar{z}, w, \bar{w}] = z + \bar{z} \quad \text{such that} \quad g[z, \bar{z}, w, \bar{w}] = -[z + \bar{z}] + 1 \leq 0$$

The problem is SFJ-univex with respect to $\eta[z, \bar{z}, z_0, \bar{z}_0] = \frac{1}{2}[z - z_0 + \bar{z} - \bar{z}_0]$,

$b_0 = b_1 = 1, \phi_0(f) = f, \phi_1 = -g$, at $z_0 = 1$, but the problem is not SFJ-invex at $z_0 = 1$ since

$$g[z_0, \bar{z}_0, w, \bar{w}] = 0 \quad \text{but}$$

$$\text{Re}[g_i(z, z, w, \bar{w})]$$

$$\geq \text{Re}[\overline{\nabla_z g_i(z_0, \bar{z}_0, w, \bar{w})} \eta^T(z, \bar{z}, z_0, \bar{z}_0) + \nabla_{\bar{z}} g_i(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)] =$$

$$-[z + \bar{z}] + 1 - \left\{ \frac{1}{2}[z - 1 + \bar{z} - 1][-1] + \frac{1}{2}[z - 1 + \bar{z} - 1][-1] \right\} = -[z + \bar{z}] + 1 + [z + \bar{z}] - 2 = -1 \leq 0$$

So the problem is SFJ-univex but not SFJ-invex .

3. Nonlinear programming :

In this section we will show that optimality and duality results still hold for the nonlinear problem (P) under weaker generalized convexity conditions.

Theorem 3.1:Optimality :



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Let $(z_0, \bar{z}_0, w, \bar{w})$ be (P) feasible for the SFJ-univex problem (P). Suppose that there

$$(u, \bar{u}, w, \bar{w}) \in C^n \times C^n \times C^m \times C^m$$

subject to

$$\text{Re}[\overline{\nabla_z f[z_0, \bar{z}_0, w, \bar{w}]} + \nabla_{\bar{z}} f[z_0, \bar{z}_0, w, \bar{w}]} + u^T \overline{\nabla_z g[z_0, \bar{z}_0, w, \bar{w}]} + u^H \nabla_{\bar{z}} g[z_0, \bar{z}_0, w, \bar{w}]}] = 0 \quad (5)$$

$$\text{Re}[g_i[z_0, \bar{z}_0, w, \bar{w}]] < 0 \Rightarrow u_i = 0, \bar{u}_i = 0, i = 1, 2, \dots, k \quad (6)$$

$$\text{and } u \geq 0, \quad (7)$$

$$\text{Further suppose that } \text{Re}[\phi_0(f_1)] \geq 0 \Rightarrow \text{Re}[f_1] \geq 0 \quad (8)$$

$$\text{Re}[f_1] \leq 0 \Rightarrow \text{Re}[\phi_0(f_1)] \leq 0 \quad (9)$$

$$\text{and that } \text{Re}[b_0(z, \bar{z}, z_0, \bar{z}_0)] > 0 \quad (10)$$

for all feasible (z, \bar{z}, w, \bar{w}) . Then $(z_0, \bar{z}_0, w, \bar{w})$ is an optimal solution of (P).

Proof :

Let (z, \bar{z}, w, \bar{w}) be (P) feasible. Then (1),(2),(5),(6),(7),(8),and (9), we have

$$\begin{aligned} & \text{Re}[b_0(z, \bar{z}, z_0, \bar{z}_0) \{f(z, \bar{z}, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})\}] \\ & \geq \text{Re}[\overline{\nabla_z f(z_0, \bar{z}_0, w, \bar{w})} \eta^T(z, \bar{z}, z_0, \bar{z}_0) + \nabla_{\bar{z}} f(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)] \end{aligned}$$



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$$\begin{aligned}
 &= \operatorname{Re}\left[\sum_{i=1}^k u_i \overline{\nabla_z g_i(z_0, \bar{z}_0, w, \bar{w})} \eta^T(z, \bar{z}, z_0, \bar{z}_0) + \sum_{i=1}^k \bar{u}_i \nabla_{\bar{z}} g_i(z_0, \bar{z}_0, w, \bar{w}) \eta^T(z, \bar{z}, z_0, \bar{z}_0)\right] \\
 &\geq -\operatorname{Re}\left[\sum_{i=1}^k b_i(z, \bar{z}, z_0, \bar{z}_0)(u, \bar{u}, w, \bar{w})\{\phi_i\{g_i(z, z, w, \bar{w})\}\}\right] \geq 0.
 \end{aligned}$$

From (10), it follows that

$$\operatorname{Re}\{\phi_0\{f(z, \bar{z}, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})\}\} \geq 0$$

By (8), we have $\operatorname{Re}[f(z, z, w, \bar{w}) - f(z_0, \bar{z}_0, w, \bar{w})] \geq 0$

Therefore, $(z_0, \bar{z}_0, w, \bar{w})$ is an optimal solution of (P).

(D) Max. $\operatorname{Re}[f(v, \bar{v}, w, \bar{w})]$

Subject

to

$$\operatorname{Re}\{\overline{\nabla_z f[v, \bar{v}, w, \bar{w}]} + \nabla_{\bar{z}} f[v, \bar{v}, w, \bar{w}]\} + x^T \overline{\nabla_z g[v, \bar{v}, w, \bar{w}]} + x^H \nabla_{\bar{z}} g[v, \bar{v}, w, \bar{w}]\} = 0$$

$$\operatorname{Re}\{[y + \bar{y}]^T g[v, \bar{v}, w, \bar{w}]\} \geq 0, \quad y \geq 0$$

Duality results can be obtained under similar conditions. We show some of them below.

Theorem 3.2 (weak duality):



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Let (x, \bar{x}, w, \bar{w}) be (P)-feasible and $\{(u, \bar{u}, w, \bar{w}), y\}$ be (D)-feasible . If there exist $\eta, \phi_0, b_0, \phi_i, b_i, i=1,2,\dots,m$, with ϕ_0 strictly increasing , such that conditions (1),(2) and (8) are satisfied at $((x, \bar{x}, w, \bar{w}), (u, \bar{u}, w, \bar{w}))$, $y_i = 0$ when $\text{Re}[g_i(u, \bar{u}, w, \bar{w})] > 0$, and

$$\sum_{i=1}^m b_i(x, \bar{x}, u, \bar{u})(y_i + \bar{y}_i)\phi_i(g_i(x, \bar{x}, w, \bar{w})) \leq 0, \text{ then } \text{Re}[f(x, \bar{x}, w, \bar{w}) - f(u, \bar{u}, w, \bar{w})] \geq 0.$$

Proof:

Assume that $\text{Re}[f(x, \bar{x}, w, \bar{w}) - f(u, \bar{u}, w, \bar{w})] < 0$. Then

$$\text{Re}[b_0(x, \bar{x}, u, \bar{u})\phi_0\{f(x, \bar{x}, w, \bar{w}) - f(u, \bar{u}, w, \bar{w})\}] < 0 \quad (11)$$

for all feasible $(x, \bar{x}, u, \bar{u}, y)$.

On the other hand , by the definition of SFJ-Univexity and the assumptions of the theorem , we

have

$$\begin{aligned} & \text{Re}[b_0(x, \bar{x}, u, \bar{u})\phi_0\{f(x, \bar{x}, w, \bar{w}) - f(u, \bar{u}, w, \bar{w})\}] \\ & \geq \text{Re}[\eta^T(x, \bar{x}, u, \bar{u})\nabla_x f(u, \bar{u}, w, \bar{w}) + \eta^T(x, \bar{x}, u, \bar{u})\nabla_{\bar{x}} f(u, \bar{u}, w, \bar{w})] \\ & = \text{Re}[\eta^T(x, \bar{x}, u, \bar{u})\sum_{i=1}^m y_i^T \nabla_x g_i(u, \bar{u}, w, \bar{w}) + \eta^T(x, \bar{x}, u, \bar{u})\sum_{i=1}^m y_i^H \nabla_{\bar{x}} g_i(u, \bar{u}, w, \bar{w})] \\ & \geq - \sum_{i=1}^m b_i(x, \bar{x}, u, \bar{u})(y_i + \bar{y}_i)\phi_i(g_i(x, \bar{x}, w, \bar{w})) \geq 0, \text{ which contradicts (9). Hence the result.} \end{aligned}$$



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Theorem 3.3 (Strong duality):

If $(x_*, \bar{x}_*, w, \bar{w})$ is (P)-feasible and a constraint qualification is satisfied at $(x_*, \bar{x}_*, w, \bar{w})$, then there exists $y_* \in C^m$ such that (x_*, \bar{x}_*, y_*) is (D)-feasible and the values of the objective functions for (P) and (D) are equal at $(x_*, \bar{x}_*, w, \bar{w})$ and (x_*, \bar{x}_*, y_*) , respectively. Furthermore, if for all (P)-feasible (x, \bar{x}, w, \bar{w}) and (D)-feasible (u, \bar{u}, y) , the hypothesis of the theorem 3.2 are satisfied, then (x_*, \bar{x}_*, y_*) is (D)-optimal.

Proof:

Since a constraint qualification is satisfied at $(x_*, \bar{x}_*, w, \bar{w})$ then there exists $y_* \in C^m$ such that the following Kuhn-Tucker conditions are satisfied.

$$\text{Re}[\overline{\nabla_{\mathbf{u}} f(x_*, \bar{x}_*, w, \bar{w})} + y_*^T \overline{\nabla_{\mathbf{u}} g(x_*, \bar{x}_*, w, \bar{w})} + \nabla_{\bar{\mathbf{u}}} f(x_*, \bar{x}_*, w, \bar{w}) + y_*^H \nabla_{\bar{\mathbf{u}}} g(x_*, \bar{x}_*, w, \bar{w})] = 0$$

$$(y_*^T + y_*^H)g(x_*, \bar{x}_*, w, \bar{w}) = 0, y_* \geq 0$$

Therefore (x_*, \bar{x}_*, y_*) is (D)-feasible.

Suppose that (x_*, \bar{x}_*, y_*) is not (D)-optimal. Then there exists (D)-feasible (u, \bar{u}, y) such that



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$\text{Re}[f(x_*, \bar{x}_*, w, \bar{w}) - f(u, \bar{u}, w, \bar{w})] < 0$. This contradicts theorem 3.2 .

Therefore (x_*, \bar{x}_*, y_*) is (D)-optimal .

4. Multiobjective Programming :

Consider the following problems :

(MP) Min $\text{Re}[f_i(x, \bar{x}, w, \bar{w})]$ such that $\text{Re}[g_j(x, \bar{x}, w, \bar{w})] \leq 0$, where

$f_i : C^n \times C^n \times C^m \times C^m \rightarrow C, i=1,2,\dots,p$, and $g_j : C^n \times C^n \times C^m \times C^m \rightarrow C, j=1,2,\dots,k$,

$m, n < p, k$, f_i and g_j are all differentiable functions and the minimum is obtained in terms of

efficiency as defined below;

(MD) Max. $\text{Re}[f_i(u, \bar{u}, w, \bar{w})]$

subject

to

$$\text{Re}[\nabla_x \{v_i^T f(u, \bar{u}, w, \bar{w}) + y_j^T g(u, \bar{u}, w, \bar{w})\} + \nabla_{\bar{x}} \{v_i^H f(u, \bar{u}, w, \bar{w}) + y_j^H g(u, \bar{u}, w, \bar{w})\}] = 0$$

$$\text{Re}[(y_i^T + y_j^H)g(u, \bar{u}, w, \bar{w})] \geq 0 \quad v \geq 0, y \geq 0.$$



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Definition 4.1: An (MP)-feasible $(x_*, \bar{x}_*, w, \bar{w})$ is said to be an efficient solution of (MP) if there exists no (x, \bar{x}, w, \bar{w}) such that $\text{Re}[f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w})] \leq 0$.

Now, we will establish optimality conditions for a point to be an efficient solution of (MP).

Theorem 4.1 (Optimality):

Let $(x_*, \bar{x}_*, w, \bar{w})$ be (MP)-feasible, suppose that there exist $v_* \in C^P, y_* \in C^k, \eta, b_0, b_1, \phi_0$ and ϕ_1 such that

$$\begin{aligned} & \text{Re}[b_0(x, \bar{x}, x_*, \bar{x}_*)\phi_0\{v_*^T(f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w}))\}] \\ & \geq \text{Re}[\eta^T(x, \bar{x}, x_*, \bar{x}_*)\{v_*^T \overline{\nabla_x f(x_*, \bar{x}_*, w, \bar{w})} + v_*^H \nabla_{\bar{x}} f(x_*, \bar{x}_*, w, \bar{w})\}] \end{aligned} \quad (12)$$

$$\begin{aligned} & \text{Re}[b_1(x, \bar{x}, x_*, \bar{x}_*)\phi_1\{y_*^T g(x, \bar{x}, w, \bar{w})\}] \\ & \geq \text{Re}[\eta^T(x, \bar{x}, x_*, \bar{x}_*)\{y_*^T \overline{\nabla_x g(x_*, \bar{x}_*, w, \bar{w})} + y_*^H \nabla_{\bar{x}} g(x_*, \bar{x}_*, w, \bar{w})\}] \end{aligned} \quad (13)$$

for all (MP)-feasible (x, \bar{x}, w, \bar{w}) and

$$\begin{aligned} & \overline{\nabla_x \{v_*^T f(u, \bar{u}, w, \bar{w}) + y_*^T g(u, \bar{u}, w, \bar{w})\}} \\ & + \nabla_{\bar{x}} \{v_*^H f(u, \bar{u}, w, \bar{w}) + y_*^H g(u, \bar{u}, w, \bar{w})\} \end{aligned} \quad (14)$$



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$$v_* \geq 0, y_* \geq 0 \quad (15)$$

$$\text{further suppose that } f \leq 0 \Rightarrow \phi_0(f) \quad (16)$$

$$\phi_1(f) \geq 0 \Rightarrow f > 0 \quad (17)$$

$$b_0(x, \bar{x}, x_*, \bar{x}_*) \geq 0, b_1(x, \bar{x}, x_*, \bar{x}_*) > 0 \quad (18)$$

for all feasible (x, \bar{x}, w, \bar{w}) . Then $(x_*, \bar{x}_*, w, \bar{w})$ is an efficient solution of (MP).

Proof:

Let (x, \bar{x}, w, \bar{w}) be (MP)-feasible, suppose that $\text{Re}[f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w})] \leq 0$. Then

$$\text{Re}[v_*^T \{f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w})\}] \leq 0 \quad \text{i.e.,}$$

Fro (14) and (16) it follows that

$$\text{Re}[b_0(x, \bar{x}, x_*, \bar{x}_*) \phi_0 \{v_*^T (f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w}))\}] \leq 0,$$

therefore by (12) $\text{Re}[\eta^T (x, \bar{x}, x_*, \bar{x}_*) \{v_*^T \overline{\nabla_x f(x_*, \bar{x}_*, w, \bar{w})} + v_*^H \nabla_{\bar{x}} f(x_*, \bar{x}_*, w, \bar{w})\}] \leq 0,$

then by (14) $\text{Re}[\eta^T (x, \bar{x}, x_*, \bar{x}_*) \{y_*^T \overline{\nabla_x g(x_*, \bar{x}_*, w, \bar{w})} + y_*^H \nabla_{\bar{x}} g(x_*, \bar{x}_*, w, \bar{w})\}] \geq 0,$

from (13), we obtain



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$\text{Re}[b_1(x, \bar{x}, x_*, \bar{x}_*)\phi_1\{(y_*^T + y_*^H)g(x, \bar{x}, w, \bar{w})\}] \geq 0$. By (17) and (18) it follows that

$(y_*^T + y_*^H)g(x, \bar{x}, w, \bar{w}) > 0$, which is a contradiction to (13) and the (MP)-feasibility of (x, \bar{x}, w, \bar{w}) . Therefore $(x_*, \bar{x}_*, w, \bar{w})$ is an efficient solution of (MP).

Theorem 4.2 (Weak duality):

Let (x, \bar{x}, w, \bar{w}) be (MP)-feasible and $\{(u, \bar{u}, w, \bar{w}), y\}$ be (MD)-feasible. If there exist η, b_0, b_1, ϕ_0

and ϕ_1 such that

$$\begin{aligned} & \text{Re}[b_0(x, \bar{x}, u, \bar{u})\phi_0\{v_*^T(f(x, \bar{x}, w, \bar{w}) - f(u, \bar{u}, w, \bar{w}))\}] \\ & \geq \text{Re}[\eta^T(x, \bar{x}, u, \bar{u})\{v_*^T \overline{\nabla_x f(u, \bar{u}, w, \bar{w})} + v_*^H \nabla_{\bar{x}} f(u, \bar{u}, w, \bar{w})\}] \end{aligned}$$

$$\begin{aligned} & \text{Re}[b_1(x, \bar{x}, u, \bar{u})\phi_1\{y_*^T g(x, \bar{x}, w, \bar{w})\}] \\ & \geq \text{Re}[\eta^T(x, \bar{x}, u, \bar{u})\{y_*^T \overline{\nabla_x g(u, \bar{u}, w, \bar{w})} + y_*^H \nabla_{\bar{x}} g(u, \bar{u}, w, \bar{w})\}] \end{aligned}$$

(16) and (17) hold and (18) is satisfied at $((x, \bar{x}, w, \bar{w}), (u, \bar{u}, w, \bar{w}))$ then

$$\text{Re}[f(x, \bar{x}, w, \bar{w}) - f(u, \bar{u}, w, \bar{w})] > 0.$$

Proof:

The proof is similar to that of Theorem 4.1.

Theorem 4.3 (Strong duality):



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If $(x_*, \bar{x}_*, w, \bar{w})$ is an efficient solution of (MP) and a suitable constraint qualification holds at $(x_*, \bar{x}_*, w, \bar{w})$ as Singh's [3] and analogous with Rueda and Mishra [13], then there exist v_* and y_* such that $((x_*, \bar{x}_*, w, \bar{w}), v_*, y_*)$ is feasible for the dual (MD) and the values of the objective functions for (MP) and (MD) are equal at $(x_*, \bar{x}_*, w, \bar{w})$ and $((x_*, \bar{x}_*, w, \bar{w}), v_*, y_*)$, respectively. Furthermore, if $v_*^T f$ and $y_*^T g$ satisfy conditions (12) and (13), ϕ_0 and ϕ_1 satisfy (16) and (17), and b_0 and b_1 satisfy (18) then $((x_*, \bar{x}_*, w, \bar{w}), v_*, y_*)$ is efficient for (MD).

Proof:

From the assumption it follows that $((x_*, \bar{x}_*, w, \bar{w}), v_*, y_*)$ is (MD)-feasible. Suppose that it is not efficient. Then there exists $((u, \bar{u}, w, \bar{w}), v, y)$ feasible such that $\text{Re}[f(x, \bar{x}, w, \bar{w}) - f(x_*, \bar{x}_*, w, \bar{w})] \geq 0$.

This contradicts Theorem 4.2. Hence, $((x_*, \bar{x}_*, w, \bar{w}), v_*, y_*)$ is efficient for (MD).

5. Conclusion:



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In this paper , we have extended an earlier work of Rueda et al. [10] to SFJ-univexity conditions .Results for minmax programming and generalized fractional programming problems can be obtained on similar lines .

Extension of this work under SFJ-pseudo-univexity in real spaces and other conditions would extend anearlier work of Kaul et al.[12]. It also extends the work of [13] .

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